Periodic motions for multy-wells potentials and layers dynamic for the vector Allen-Cahn equation

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Abstract

Let $W : \mathbb{R}^m \to \mathbb{R}$ a potential that satisfies

$$0 = W(a) < W(u), \ a \in A, \ u \in \mathbb{R}^m \setminus A,$$

where $A \subset \mathbb{R}^m$ is a finite set with at least two distinct elements. We consider the Hamiltonian system

$$u'' = W_u(u), \ W_u(u) = \left(\frac{\partial W}{\partial u_1}(u), \ldots, \frac{\partial W}{\partial u_m}(u)\right)^\top,$$

and, given a small number $\delta > 0$ and $N \geq 2$ points $a_1, \ldots, a_N \in A$ with $a_j \neq a_{j+1}$, $j = 1, \ldots, N-1$ and $a_N \neq a_1$ we study the existence of periodic solutions $u_\delta : \mathbb{R} \to \mathbb{R}^m$ that satisfies

$$|u_\delta(s_j^\delta) - a_j| < \delta, \ \ j = 1, \ldots, N,$$

for some $s_1^\delta < \ldots < s_N^\delta \in [0, T^\delta)$, $T^\delta > 0$ the period of u^δ.

We also discuss the problem of layers dynamics for the vector Allen-Cahn equation

$$u_t = \epsilon^2 u_{xx} - W_u(u), \ \ x \in (0, 1),$$

with periodic boundary conditions.